

TEXTILE ECONOMICS AND COSTING

CHAPTER 5: ECONOMIC ANALYSIS OF INDUSTRIAL OPERATION

Definitions Pertaining to Costs

- Every industrial operation must be carefully examined to obtain maximum **economic efficiency of an operation**.
- let X denote variable that strongly influences the cost of an operation e.g. the number of units of a **standard commodity** that a firm produces per month, **units of production of yarn, fabric** per day.
- cost that remains **constant** during this change is a **fixed cost** with respect to X , and one that **changes** is a **variable one**

E.g. **rent** paid for the factory is a **fixed cost**, and the cost of the **raw materials** is a **variable cost**.

Cont.d

- Intermediate between these extremes are *semi-variable* costs, which are composite costs that contain both fixed and variable elements. E.g. *cost of electric power*
- ❖ Variable costs can often be divided into three subgroups:
 - *directly varying costs*- directly proportional to X (*royalty payment*)
 - *inversely varying costs* - inversely proportional to X (*unit cost*)
 - those that vary in some more complex manner (Standard cost)

Cont.d

- *royalty payment* –an agreement that stipulates the inventor to receive a fixed sum for each unit manufactured from the use of patent
- *unit cost* obtained by dividing the *total cost* by *total number of units*.
- *Standard costs* are pre-established costs that are designed to *gauge the efficiency* of an operation.

Cont.d

❖ The two widely used methods of securing standard costs are:

1. Obtaining them statistically on the basis of **historical data**. This method has serious deficiencies.

- ✓ First, if the operation was **performed poorly in the past**, the use of these standard costs may serve to perpetuate inefficiency.
- ✓ Second, **new technology** may permit a reduction in costs, thereby rendering the historical data irrelevant

Cont.d

2. Equating level of efficiency to currently attainable costs (the preference of many analysts)
 - This is performance judgmental way, whereas the standard cost based on historical data is an objective one.

Minimization of Cost

- Let C denote the **total cost** of a project or operation, and assume that X is the only **variable** that influences C . Also assume that C is composed exclusively of directly varying costs, inversely varying costs, and fixed costs. Then,

$$C = aX + (b/X) + c = aX + bX^{-1} + c$$

where a , b , and c are constants

Cont.d

- Now let $C_{\min}$ = the minimum value of C and X_0 = value of X corresponding to $C_{\min}$
- Setting the derivative dC / dX equal to zero and solving for X , we obtain: $\frac{dC}{dX} = a - \frac{b}{X^2}$ $0 = a - \frac{b}{X^2}$
- $X_0 = \sqrt[3]{(b/a)}$
- Thus, X_0 is independent of fixed costs.

Cont.d

- Applying the expression for X_o , we find that

$$\text{Kelvin's law} \dots aX_o = (b/X_o) = \sqrt{ab}$$

- This equation reveals that C is minimum when the **sum of the directly varying costs equals the sum of the inversely varying costs**, and this interesting relationship is known as *Kelvin's law*. It follows that

$$C_{\min} = 2(\sqrt{ab}) + c$$

Example 5.1

- A steel bridge is to have a length of 420m. The cost of the steel and of an individual pier are estimated to be as follows: $C_s = 1,200,000 + 47,000X$ and $C_p = 840,000 + 560X$, where C_s is the cost in dollars of the steel, C_p is the average cost in dollars of a pier, and X is the span in meters. All other costs are independent of the span. Determine the most economical span for this bridge.

$P =$ pier

$P = n+1$, n is number of span

solution

- The number of piers is $420/X + 1$. Therefore, the total cost of the piers is:

$$([420/X]+1)(840,000+560X)=352,800,000/X+560X+1,075,200$$

- By summation, the total cost C is:

$$C = C_s + C_p$$

$$= 1,200,000 + 47,000X + 352,800,000/X + 560X + 1,075,200$$

$$= 47,560X + 352,800,000/X + 2,275,200$$

$$X_o = \sqrt{(352,800,000/47,560)} = 86.1 \text{ m}$$

Cont.d

- Number of piers required = $(420/86.1) + 1 = 5.88$
- When number of pier is six $C_p = 843,360$
- When number of pier is five $C_p = 842,800$
- This number must of course be an integer. making $X_o = 420/5$
 $= 84m$.

Location of Break-even Points

- Break-even point is a point at which **two lines intersect**; alternatively, it is a point, where two variables are equal.
- Break-even point is the number of units of production which a firm must produce and sell in order to merely pay for the fixed costs
- Beyond the break-even point, the firm earns a profit
- For example, the term may refer to
 - ✓ the point at which **revenue from the sale of a commodity is equal to the cost of producing the commodity**, or
 - ✓ the point at which the **cost of production is identical under two alternative methods of manufacture.**

Example 5.2

Two companies, A and B, manufacture the same commodity. Company A uses a **mechanized process**, and company B relies mainly **on manual labor**. The fixed cost is \$40,000 per month for A and \$15,000 per month for B. The indirectly varying cost is \$14 per unit for A and \$52 per unit for B. The selling price is \$85 per unit for each company.

- a. At what volume of production are the unit costs of the two companies identical?
- b. How many units must each company sell each month merely to avoid a loss?

solution

Let

X = number of units produced and sold per month

C = total cost of producing X units

U = unit cost

P = monthly profit

Using a subscript to identify the company,

$$C_A = 40,000 + 14X \quad (a)$$

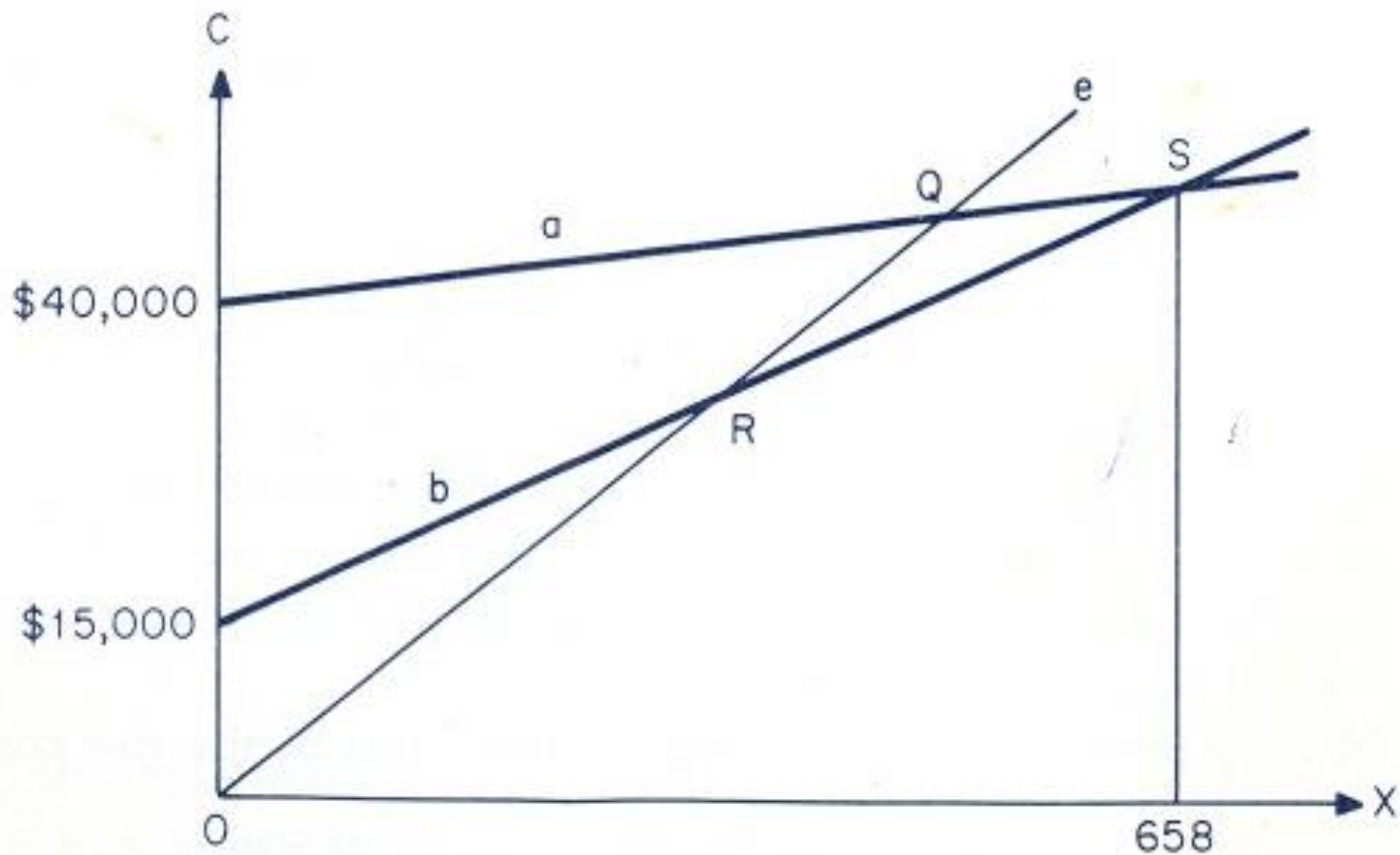
$$C_B = 15,000 + 52X \quad (b)$$

$P = S - C = SX - C$, Where, $S = SX$

$$P_A = 85X - (40,000 + 14X) = 71X - 40,000 \quad (c)$$

$$P_B = 85X - (15,000 + 52X) = 33X - 15,000 \quad (d)$$

Part *a*: In Fig. below lines *a* and *b* represent Equations (*a*) and (*b*), respectively, and *e* is an arbitrary straight line through the origin.



Cont.d

- Since $U = C/X$, it follows that:

$$U_A \text{ at } Q = U_B \text{ at } R = \text{slope of } e$$

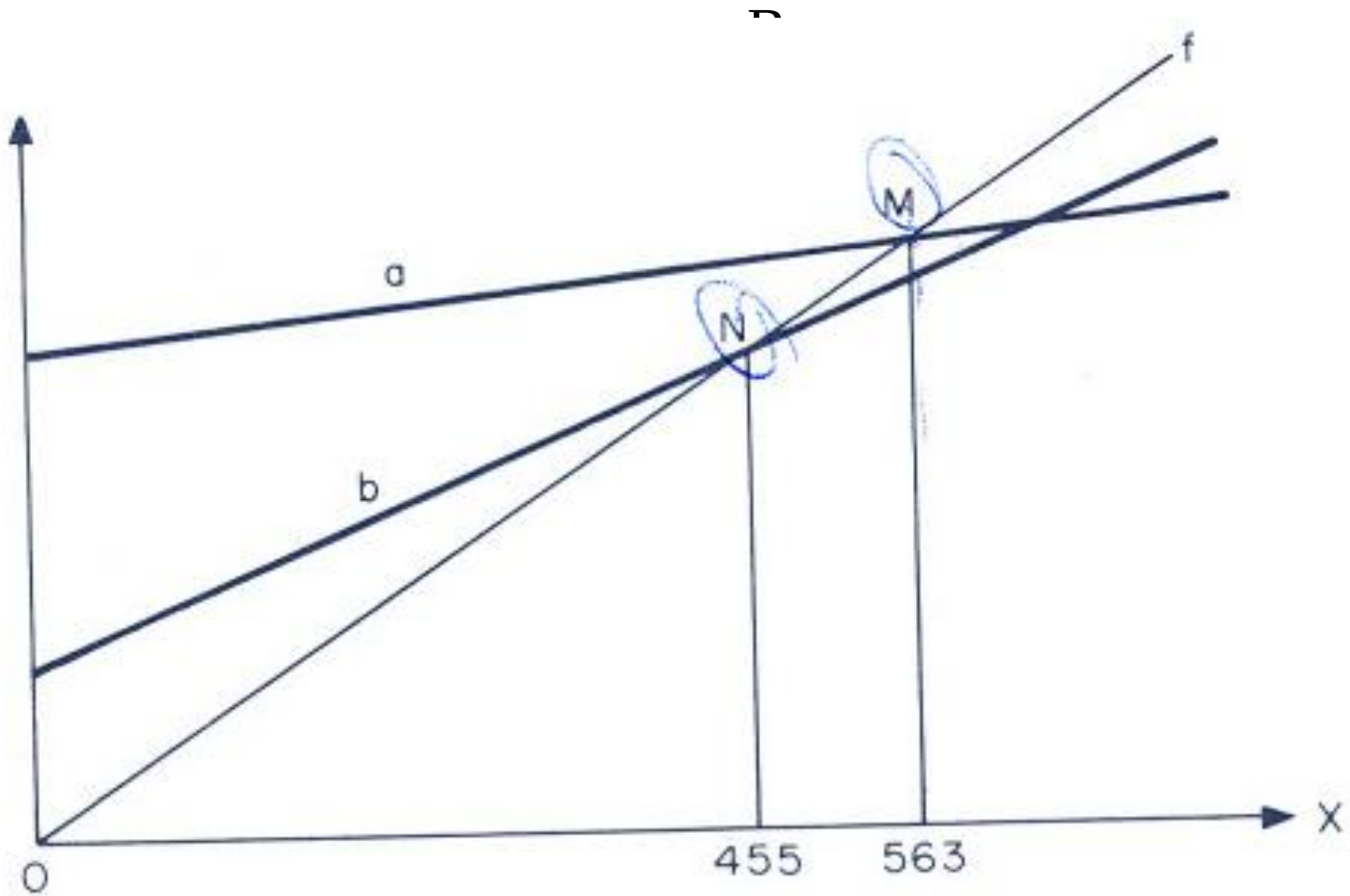
- Manifestly, the unit costs are equal at the point S where lines a and b intersect.
- At this point,

$$40,000 + 14X = 15,000 + 52X$$

$$\text{Solving, } X = 658 \text{ units/month}$$

- Point S in the Fig. is a **break-even point** in the respect that it reveals the minimum production that is needed to justify use of the mechanized process.

Part *b*: Refer to Fig. below, Lines *a* and *b* again represent Equations (*a*) and (*b*), respectively, and line *f* represents the income from sales, which is $85X$. The profit is zero at point M for company A and at point N for



Cont.d

- Applying Equations (c) and (d), we obtain the following results:

$$\text{At M: } 71X - 40,000 = 0 \quad \Rightarrow \quad X = 563 \text{ units/month}$$

$$\text{At N: } 33X - 15,000 = 0 \quad \Rightarrow \quad X = 455 \text{ units/month}$$

- Thus, companies A and B must sell 563 and 455 units per month, respectively, merely to avoid a loss.
- Points M and N are **break-even points** in the respect that they reveal the minimum number of units each firm must produce and sell to cover expenses.

Analysis of Profit

Assume the following:

- A firm produces and sells a standard commodity;
- the firm can sell as many units of this commodity as it produces, and
- the selling price remains constant.

Let X = number of units produced per period

C = total cost of production per period

S = sales revenue per period

P = profit per period

s = selling price.

i = investment rate

Cont.d

Then $S = sX$

$$P = S - C = sX - C$$

$$i = P/C = (S/C) - 1$$

- For a given value of X , the *incremental* (or *marginal*) cost is the cost of producing one additional unit.
- When X is large, incremental cost $= \frac{dC}{dX}$
- Unit cost $= \frac{C}{X}$

Cont.d

- incremental investment rate is the investment rate earned by producing one additional unit.
- Let c and i_i denote the incremental cost and incremental investment rate, respectively. Then,

$$i_i = (s - c)/c = (s/c) - 1$$

$$\text{OR } i_i = [s/(dC/dX)] - 1$$

- the value of c tends to increase as X increases. This tendency is referred to as the *law of diminishing returns*

Cont.d

- The maximum profit is attained at the point where c becomes equal to s ; beyond this point, the cost of producing additional units exceeds the income from those units
 - However, when the firm wishes to earn a certain minimum acceptable rate of return (MARR), production should be halted before the point of maximum profit is reached
- ❖ there are four points on the profit curve that are of particular significance:
- the point of zero profit,

Cont.d

- the point of maximum profit,
- the point of maximum investment rate, and
- the point of optimal production, which is the point at which i_i becomes equal to the MARR.
- **Point of maximum profit**

By setting $dP/dX = 0$, we obtain $dC/dX = s$

OR Incremental cost = selling price

- **Point of maximum investment rate**

By setting $di/dX = 0$, we obtain $dC/dX = C/X$

Cont.....

OR Incremental cost = unit cost

Point of optimal production

Let q denote the MARR. Setting $i_i = q$, *we obtain*

$$dC/dX = s/(1+q)$$

Example 5.3

A firm produces a standard commodity that it sells for \$450 per unit. The monthly cost of production in dollars is estimated to be

$$C = 130X + 0.26X^2 + 56,000$$

where X denotes the number of units produced per month.

Locate the point of **zero profit**, the point of **maximum profit**, the point of **maximum investment rate**, and the point of **optimal production** as based on a 12 percent MARR. Compute the investment rate at each of the last three points.

Cont.d

$$P = S - C = 450X - (130X + 0.26X^2 + 56,000)$$

$$P = 320X - 0.26X^2 - 56,000 = ax^2 + bx + c$$

$$dC/dX = 130 + 0.52X$$

$$C/X = 130 + 0.26X + (56,000/X)$$

- ***Point of zero profit (A):***

Setting $P = 0$ and solving for X , we obtain $X = 211$ units and $X = 1020$ units. Only the first value of X is significant in the present context.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Cont.d

- *Point of maximum profit (B):*

$$Dc/Dx = s \quad 130 + 0.52X = 450 \quad X = 615 \text{ units}$$

At this value of X , $C = \$234,289$ and **$P = \$42,461$**

$$i = 42,461/234,289 = 18.1 \text{ percent}$$

- *Point of maximum investment rate (C):*

$$dC/dX = C/X, \quad 130 + 0.52X = 130 + 0.26X + (56,000/X)$$

$$X = 464 \text{ units}$$

- At this value of X , $C = \$172,297$ and $P = \$36,503$

$$\mathbf{i_{\max} = 36,503/172,297 = 21.2 \text{ percent}}$$

Cont.d

- *Point of optimal production (D):*

$$dC/dX = s/(1+q) \quad 130 + 0.52X = 450/1.12$$

$$X = 523 \text{ units}$$

- At this value of X , $C = \$195,108$ and $P = \$40,242$

$$i = 40,242/195,108 = 20.6 \text{ percent}$$

four pt on the profit curve	Value of X	Cost	Profit	investment rate%
<i>zero profit</i>	211	95,005.46	0	0
<i>maximum profit</i>	615	234,289	42,461	18.1
<i>Max. investment rate</i>	464	172,297	36,503	21.2
<i>optimal production</i>	523	195,108	40,242	20.6

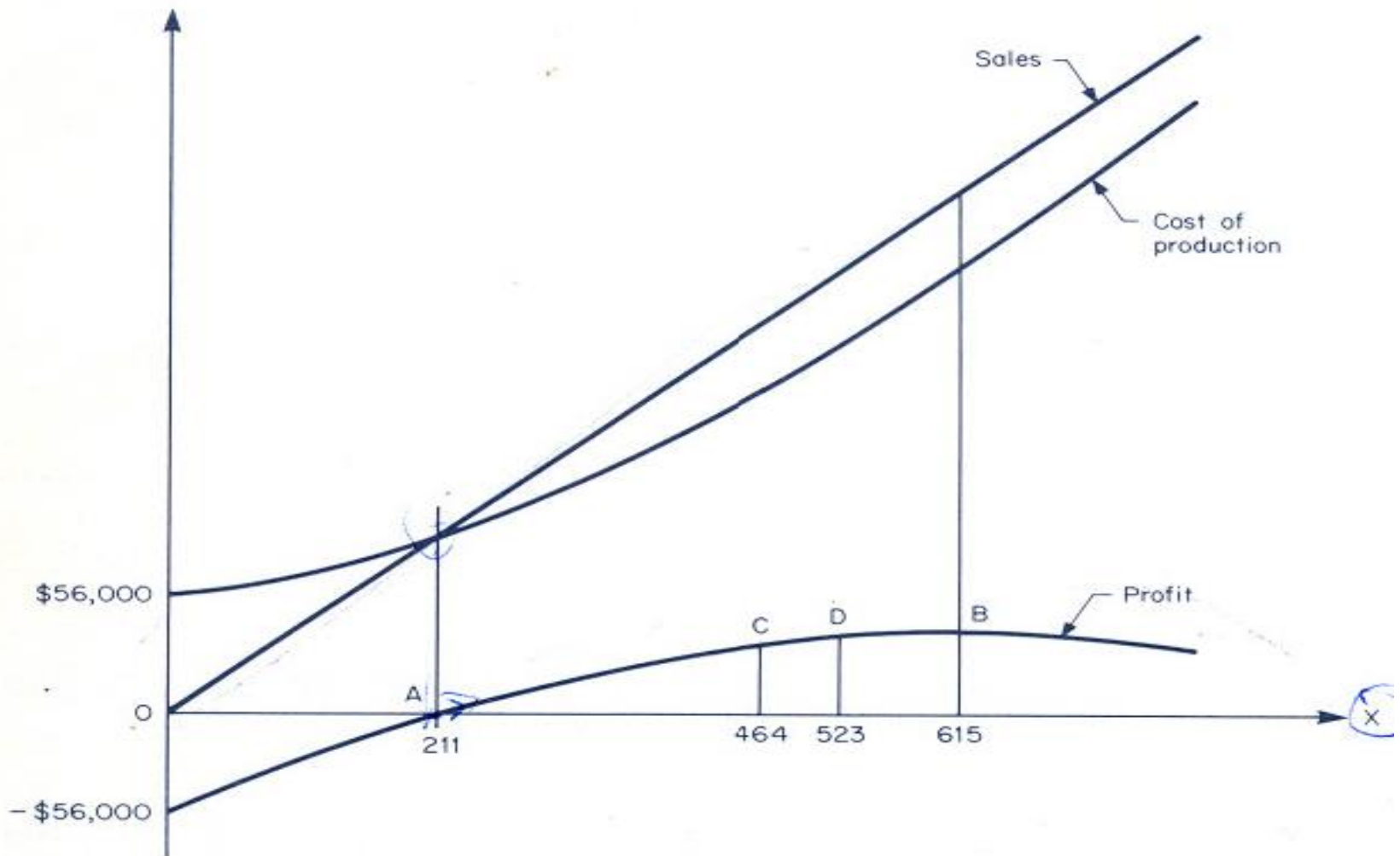


Fig. Variation of profit with production